



Contingent convertible bonds with the default risk premium

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ABSTRACT

Contingent convertible bonds (CoCos) are hybrid instruments characterized by both debt and equity. CoCos are automatically converted into equity or written down when a predefined trigger event occurs. The present study quantifies the issuing bank's default risk that only manifests in the post-conversion period for pricing CoCos depending on a loss-absorbing method. This work aims to reflect the distinct features of equity-conversion CoCos - in contrast to a write-down CoCos - in a valuation framework. Accordingly, we propose a model to compute the ratio of common equity Tier 1 (CET1), which is composed of core capital and risky assets, by employing a geometric Brownian motion and a random variable. Then, we formulate the post-conversion risk premium by measuring the probability with which the bank's CET1 ratio breaches a regulatory default threshold after conversion. Finally, we empirically examine a positive value of the post-conversion risk premium embedded in the market prices of equity-conversion CoCos.

1. Introduction

During the global financial crisis of 2007–2008, many financial institutions were left severely under-capitalized. Some major banks were bailed out by taxpayers rather than bailed in by creditors because the old-style subordinated debts had failed to act as a buffer against losses during times of distress. Substantial government intervention and financial support were necessary to prevent many banks from becoming insolvent, resulting in a need for stronger regulation of the banking system. As a part of the revised banking regulation, Basel III has implemented strict capital requirements to enhance banks' financial stability and reduce systemic risk (Basel Committee on Banking Supervision, 2011). The major changes in Basel III place more weight on core capital, i.e. common equity Tier 1 (CET1). The minimum requirement level for CET1 capital, a newly established category for banks' capital structure, was gradually phased in up to 4.5% of total risk-weighted assets (RWA) until 2015.

One remarkable evolution in the capitalization of banks under this new regulation is the emergence of a new hybrid asset class called *contingent convertible bonds* or CoCos for short. CoCos are a type of bond that is automatically converted into equity or written down when the issuer's capital-ratio falls below a specified level. This automatic conversion characteristic means that CoCos are expected to reduce the economic costs of bankruptcy for the benefit of all debt and equity

holders. According to Basel III, CoCos are eligible capital instruments for meeting buffers (see European Banking Authority, 2011) because they may help reduce bank vulnerability and provide greater counter-cyclical resilience. The combination of the regulatory environment and the pressure on banks to recapitalize has led rapid growth of the CoCo market over the past decade. The global issuance of CoCos was estimated to be USD 360 billion until 2015 since the first issue by the Lloyds Banking Group in 2009 (Fig. A.4).

Despite high demand for CoCos in the financial industry, modelling and pricing CoCos are still challenging issues because the equity and credit risk are incorporated into a single product. For the design of CoCos, Flannery (2005, 2009) and Pennacchi, Vermaelen, and Wolff (2014) introduce 'reverse convertible debentures' and 'call option enhanced reverse convertibles', respectively, as examples of the structure of early CoCo proposals. McDonald (2013) suggests that CoCos with a dual-trigger that depends on the situation of both the individual firm and the whole banking system. Sundaresan and Wang (2015) discuss on stock price trigger CoCos and the nonexistence of a unique equilibrium in their prices.

On the valuation of CoCos, one strand of the literature is based on structural bond pricing models (e.g. Leland, 1994). The value of CoCos can be derived as an optimal level when firms' capital structure is composed of equity, subordinated debt, and CoCos (Pennacchi, 2011; Glasserman & Nouri, 2012; Brigo, Carcia, & Pede, 2015; Albul, Jaffee, &

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Tchisti, 2015; Yang & Zhao, 2015; Chen, Glasserman, Nouri, & Pelger, 2017). The other strand of literature on the valuation of CoCos uses the pricing techniques of financial derivatives from two viewpoints, that of equity and fixed-income derivatives, as proposed by De Spiegeleer and Schoutens (2010, 2012). Cheridito and Xu (2015) apply a reduced-form approach, and Corcuera et al. (2013) employ a Lévy process to model CoCos. Chung and Kwok (2016) evaluate capital-ratio trigger CoCos including a discretionary conversion based on two-dimensional stochastic processes. Corcuera et al. (2014) derive a closed-form formula of coupon-cancellable CoCos, a new type of CoCo where coupon can be cancelled during the contract. Among empirical approaches, De Spiegeleer, Hocht, Marquet, and Schoutens (2017) estimate the implied volatility of the CET1 ratio by using the market prices of issued CoCos, and Wilkens and Bethke (2014) conduct a comparative analysis via a hedging simulation to assess the best-fitting model in practice.

In this study, we propose a novel model of the CET1 ratio, which is defined as the ratio of a bank's equity value to its RWA value for an in-depth analysis in CoCos by building a hidden random threshold. In our setup, we assume that the equity price follows a geometric Brownian motion and that bank's RWA is unknown on the evaluation date, but only its distribution is given. Although the true value of a bank's RWA is revealed at the time of conversion, it can be progressively estimated by using available balance sheet information at issuance. Under this model, we then derive the theoretical prices of CoCos with a CET1 ratio trigger and their expected recovery rates in an analytic form.

In addition, we develop a valuation framework to examine the difference between CoCos with two loss-absorbing mechanisms: equity-conversion (EC) and write-down (WD). To address distinctive risk that can exist only in EC CoCos, we introduce two new concepts: *regulatory default* and *post-conversion risk*. We define regulatory default as the likelihood that an issuing bank fails to retain the minimum capital requirement set by regulators and post-conversion risk as the likelihood that regulatory default occurs in the ex-post conversion situation. We derive the premium caused by post-conversion risk that only manifests after conversion by computing the price difference between two CoCos with/without post-conversion risk.

The main contributions of this study are twofold. First, we introduce a methodology for pricing EC CoCos that are distinguished from the WD type by using a new CET1 ratio model. Many studies discuss whether issuing CoCos can improve financial stability and, if so, the extent to which this can reduce systemic risk in the entire banking system (see Jaworski, Liberadzki, & Liberadzki, 2017; Hilscher & Raviv, 2014). However, studies of the difference between the loss-absorbing methods of CoCos are scant - in the context of banks' incentive to issue (or investors' risk to take) EC CoCos in contrast to WD CoCos. Among this limited literature, Avdjiev, Bolton, Jiang, Kartasheva, and Bogdanova (2015) empirically examine how the issue announcement affects each type of CoCo, while Martynova and Perotti (2018) present a theoretical approach for different risk-taking incentives in a discrete time setup. Meanwhile, the present study provides theoretical and empirical frameworks that differentiate between the method of valuation for EC and WD CoCos over a continuous time period by quantifying post-conversion risk. Second, we present empirical evidence that the post-conversion risk premium is constantly reflected in the real market prices of EC CoCos. By adopting a rigorous procedure for our empirical tests, we estimate a post-conversion risk premium of around 2% charged on the EC CoCos issued by Credit Suisse. This result bridges the gap between our theoretical proposal and the ongoing market perception of the loss-absorbing functions of CoCos.

The rest of the paper is organized as follows. Section 2 proposes a model for CoCos with a CET1 ratio trigger with several types of conversion and loss-absorbing methods in a single formula. Section 3 derives the closed-form formulae of CoCos with a CET1 ratio trigger and their expected recovery rates. Section 4 presents a model for post-conversion risk and shows a method of quantifying the post-conversion risk premium by employing a compound barrier option pricing idea.

Section 5 estimates the post-conversion risk premium from the real market prices of CoCos. Section 6 concludes. The appendix includes the technical proofs, additional numerical tests, and figures and tables.

2. Model for contingent convertibles

The CoCo conversion process is activated when a certain identifier breaches a specified level. CoCos have two defining characteristics: (i) a trigger that activates conversion and (ii) a loss-absorbing mechanism that specifies how losses are absorbed at conversion.

Two types of triggers are mainly employed in practice: the capital-ratio trigger and regulatory trigger. The capital-ratio trigger is set based on accounting values in balance sheets such as equities and liabilities which makes it easy to show the overall capital sufficiency of banks with the one drawback that information on capital-ratios is not continuously available because of infrequent updates. The regulatory trigger is implemented based on a regulator's judgement on the solvency prospects of issuing banks. This trigger is controlled by authorities, which makes it difficult to quantify the probability of conversion. Once conversion is activated under the defined trigger, a loss-absorbing process is automatically enforced in two directions: a bond principal is either converted into common equity (EC-type) or written off (WD-type).

In this study, we focus on a CET1 ratio trigger. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space, and $(\mathcal{F}_t)_{t \geq 0}$ be a natural filtration generated by a Brownian motion $(W_t)_{t \geq 0}$. Assume that an equity price process S_t under an equivalent risk-neutral martingale measure \mathbb{Q} , satisfies

$$dS_t = rS_t dt + \sigma S_t d\tilde{W}_t, \quad (1)$$

where \tilde{W}_t is a \mathbb{Q} -Brownian motion, r is the risk-free interest rate, and σ is the volatility of equity price S_t .

The CET1 ratio is formulated to a ratio of the bank's CET1 capital to its total RWA amount. Under Basel III, a bank's capital is categorized into three levels: CET1, Additional Tier 1, and Tier 2 capital. Among them, CET1 capital includes common shares issued by a bank, stock surplus, and retained earnings.¹ Meanwhile, total RWA is calculated as the weighted sum of the risk exposures of credit, market, and operational risky assets. To assess each risk position, either a linear weighting scheme or a value-at-risk approach is used. A linear weighting scheme assigns different weights depending on the level of risk. A value-at-risk approach computes the expected losses within a time horizon under a certain confidence level.² According to Le Leslé and Avramova (2012), credit risk is the largest component of total RWA, representing 86% on average, while market and operational risks account for 6.5% and 7.5%, respectively.

By the definition of each portion, the CET1 ratio is set as follows:

$$\text{CET1 ratio} = \frac{\text{CET1 capital}}{\text{Total RWA}} \approx \frac{S_t \times M}{\text{Total RWA}} = \frac{S_t}{\text{Total RWA}/M}, \quad (2)$$

where M is the total number of shares issued by a bank and S_t is the share price at time t , as defined in Eq. (1).

Let us define L as the RWA-per-share value of a CoCo-issuing bank, which is the total RWA amount divided by the number of shares that a bank issued, i.e. $\text{Total RWA}/M$. We assume that L is a non-negative random variable with distribution F , which is independent of filtration $(\mathcal{F}_t)_{t \geq 0}$, and $(\mathcal{G}_t)_{t \geq 0}$ is an enlarged filtration, defined by $\mathcal{G}_t = \mathcal{F}_t \vee \sigma(L)$.

According to the definition of the CET1 ratio, conversion time τ_B can be represented as the first time when S_t/L falls below a threshold value α_0 :

¹ Additional Tier 1 capital consists of non-cumulative preferred stock, and Tier 2 capital includes debt subordinated to depositors with an original maturity of five years.

² A one-year 99.9% confidence interval is given for credit risk and a 10-day 99% confidence interval is given for market risk.

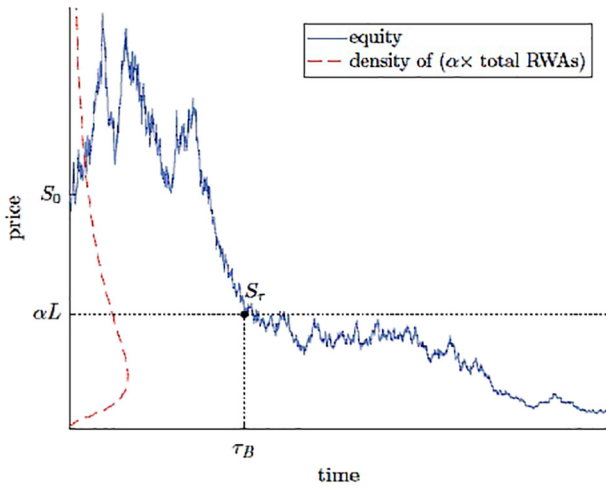


Fig. 1. Description of the proposed CET1 ratio model and conversion time τ_B under the model.

$$\tau_B = \inf \left\{ t \geq 0 : \frac{S_t}{L} \leq \alpha_0 \right\}. \quad (3)$$

Hence, this is the first hitting time when equity price S_t reaches random barrier $\alpha_0 L$. The random barrier is hidden and only its distribution is known; a single value is revealed at the time of conversion. We extend the equity derivative approach (De Spiegeleer & Schoutens, 2010) by adding a random barrier for the conversion time. τ_B is the stopping time with respect to an enlarged filtration \mathcal{G}_t but not \mathcal{F}_t . The exact value L is unobservable during time $0 \leq t < \tau_B$ and is only revealed at conversion time $t = \tau_B$.

Fig. 1 describes the proposed idea. This model captures the uncertainty of a CET1 ratio by letting L be a random variable. The true level of L does not evolve over time and its exact value cannot be observed on a daily-basis. Random trigger barrier $\alpha_0 L$ can be hit unexpectedly by equity price S_t , resulting in a jump-like event.

At conversion time τ_B , a proportion of the CoCo's principal N is absorbed into the capital layer of the issuing bank, and the rest is paid to the CoCo holder. For EC-type CoCos, the number of converted equities is called the conversion ratio C_r , and this is determined by conversion price C_p . C_p is set in a CoCo contract and specified at the issuing date as

$$C_p(S_{\tau_B}) = \max(\beta S_{\tau_B}, S^*), \quad (4)$$

with constants $0 \leq \beta \leq 1$ and $S^* \geq 0$. Two types of conversion prices are mainly used when designing CoCos: (i) Fixed conversion price: If $\beta = 0$, then $C_p(S_{\tau_B}) = S^* = \gamma S_0$, which is specified at the issue date for constant γ , (ii) Floor conversion price: If $\beta = 1$ and $S^* = S_F > 0$, then $C_p(S_{\tau_B}) = \max(S_{\tau_B}, S_F)$. For WD-type CoCos, a CoCo's principal is wiped out and is only recovered with a proportion δ specified in the issuance. The payment amount of WD CoCos is δN .

Consider a zero-coupon CoCos with face value N and maturity T . The contractual payment upon the trigger can be formulated as follows. Let w be the proportion of N for EC, $0 \leq w \leq 1$, and $(1 - w)$ be the proportion of N for WD with a recovery rate of $0 \leq \delta \leq 1$.³ If a trigger occurs before maturity T , the amount $\bar{K}(S_{\tau_B})$ is paid at conversion τ_B :

$$\bar{K}(S_{\tau_B}) = wN \frac{S_{\tau_B}}{C_p(S_{\tau_B})} + (1 - w)\delta N = C_r(S_{\tau_B}) \times S_{\tau_B} + (1 - w)\delta N, \quad (5)$$

where $C_r(\cdot)$ is given by $wN/C_p(\cdot)$; otherwise, the full face value N is paid at maturity T . Hereafter, we call $\bar{K}(S_{\tau_B})$ the 'contractual payoff'.

3. Valuation of CoCos

In this section, we present a formula for calculating the theoretical price of CoCos with a CET1 ratio trigger in the model proposed in Section 2. Finding CoCo prices determining the extent to which a coupon is enhanced to reflect the likelihood of conversion compared with a conventional bond. The main idea for pricing is based on a risk-neutral valuation framework.

The following result shows the probability distribution of the first passage of time reaching a fixed barrier for a geometric Brownian motion.

Remark 1 (Shreve, 2004). Let S_t satisfy Eq. (1). Define stopping time τ as the first time at which S_t reaches a fixed barrier $B < S_0$. Then, the distribution H of τ , $\mathbb{E}[1(\tau \leq t)]$, is given by

$$H(t; B, S_0) = \Phi \left(\frac{\ln(\frac{B}{S_0}) - (r - \frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}} \right) + \left(\frac{B}{S_0} \right)^{\frac{2r}{\sigma^2} - 1} \Phi \left(\frac{\ln(\frac{B}{S_0}) + (r - \frac{1}{2}\sigma^2)t}{\sigma\sqrt{t}} \right), \quad (6)$$

where Φ is the cumulative distribution function of the standard normal variable. Furthermore, the probability density function h of stopping time τ is given by

$$h(t; B, S_0) = -\frac{\ln(\frac{B}{S_0})}{\sqrt{2\pi\sigma^2 t^3}} \exp \left(-\frac{\left[\ln(\frac{B}{S_0}) - (r - \frac{1}{2}\sigma^2)t \right]^2}{2\sigma^2 t} \right).$$

Eq. (3) implies that the equity price at trigger time, namely $S_{\tau_B} = \alpha_0 L$, is given by a random variable that depends on RWA-per-share level L .

Definition 2. Let \mathcal{F}_t be the filtration generated by a stock price up to time t . (i) A random variable $\tau_B^x: \Omega \rightarrow [0, \infty]$ is conversion time τ_B when RWA-per-share level L is set to $x \geq 0$, i.e.

$$\tau_B^x = \inf\{t: S_t \leq \alpha_0 x\}$$

with a CET1 threshold α_0 .

(ii) Functions $G_t(\cdot; x)$ and $g_t(s; x)$ are defined as the conditional distribution and the conditional probability density of τ_B^x , seen from time t , respectively, i.e.

$$G_t(s; x) = \mathbb{Q}(\tau_B^x \leq s | \mathcal{F}_t), \quad g_t(s; x) = \mathbb{Q}(\tau_B^x \in [s, s + ds] | \mathcal{F}_t),$$

where \mathbb{Q} is a risk neutral martingale measure for $s \geq t$.

(iii) A function $G_t(\cdot)$ is defined as the conditional distribution of conversion time τ_B defined in Eq. (3), as seen at time t , i.e. for $s \geq t$,

$$G_t(s) = \mathbb{Q}(\tau_B \leq s | \mathcal{F}_t) = \int_0^\infty \mathbb{Q}(\tau_B^x \leq s | \mathcal{F}_t) dF(x),$$

where F is the cumulative distribution of L .

The next lemma shows the formula of the conditional distribution of conversion time τ_B^x .

Lemma 3. For $s \geq t$, the conditional distribution $G_t(s; x)$ of conversion time τ_B^x and $G_t(s)$ of conversion time τ_B are given by

$$G_t(s; x) = 1(\tau_B^x \leq t) + 1(\tau_B^x > t)H(s - t; \alpha_0 x, S_t),$$

where $H(t; \cdot, \cdot)$ is given in Eq. (6), and

$$G_t(s) = 1 - F \left(\frac{m_t}{\alpha_0} \right) + \int_0^{\frac{m_t}{\alpha_0}} H(s - t; \alpha_0 x, S_t) dF(x), \quad (7)$$

where $m_t = \min_{0 \leq v \leq t} S_v$ is a running minimum. $G_t(s)$ represents the probability with which a CET1 ratio reaches α_0 to time s when stock price S_u has been investigated for time $0 \leq u \leq t$. If a current stock

³ We assume that the recovered amount is paid out in cash.

price S_t is greater than barrier $B = \alpha_0 x$, then the probability density of τ_B^x is well defined. If S_t is equal to or less than this barrier, the probability density of τ_B^x is concentrated on a single point, t .

The following theorem shows the formula for zero-coupon CoCos that pay the contractual payoff in Eq. (5) at conversion.

Theorem 4 (Zero-coupon CoCos). Suppose that a zero-coupon CoCo has a unit face value and maturity T . Let $\bar{K}(\cdot)$ be the contractual payoff in Eq. (5) with $N = 1$, and $D(t, s)$ be a discount factor with a constant risk-free rate r . Then, the value of the zero-coupon CoCo at time $0 \leq t \leq T$ is given by

$$\begin{aligned} \bar{P}^{ZC}(t, T) = & \int_{\frac{S_0}{\alpha_0}}^{\frac{S_0}{m_t}} \bar{K}(\alpha_0 x) D(t, \tau_B^x) dF(x) + \frac{\bar{K}(S_0)}{D(0, t)} \left\{ 1 - F\left(\frac{S_0}{\alpha_0}\right) \right\} \\ & + \int_0^{\frac{m_t}{\alpha_0}} \bar{K}(\alpha_0 x) \int_t^T D(t, s) h(s - t; \alpha_0 x, S_t) ds dF(x) \\ & + D(t, T) \{1 - G_t(T)\}, \end{aligned}$$

where $G_t(T)$ is given in Eq. (7) and $m_t = \min(0 \leq v \leq tS_v)$ is the running minimum.

CoCo investors can receive coupons only if conversion does not occur, which is identical to a conventional bond bearing coupons.

Corollary 5 (Coupon-bearing CoCos). Suppose that a CoCo pays n coupons of amount c_i at each coupon date t_i based on a unit face value $N = 1$. Each coupon c_i is paid until when conversion τ_B does not occur before maturity $T = t_n$. The discounted payoff of the coupon-bearing CoCo at time t is given by

$$\begin{aligned} 1(\tau_B \leq T) \left[\sum_{i=1}^n c_i D(t, t_i) 1(\tau_B > t_i) + \bar{K}(S_{\tau_B}) D(t, \tau_B) \right] \\ + 1(\tau_B > T) \left[\sum_{i=1}^n c_i D(t, t_i) \right]. \end{aligned}$$

Therefore, the value of a coupon-bearing bond is represented as the sum of zero-coupon CoCos with maturity t_i and principal c_i for $1 \leq i \leq n$, and a zero-coupon CoCo with maturity $T = t_n$ and principal $N = 1$. The value of coupon-bearing CoCos is given by

$$\bar{P}^{CB}(t, T) = \sum_{i=1}^n c_i \bar{P}^{ZC}(t, t_i) + \bar{P}^{ZC}(t, T),$$

where $\bar{P}^{ZC}(t, t_i)$ is given in Theorem 4.

The recovery rates of WD CoCos are fixed at issuance, $R_{WD} = \delta$. However, the recovery rates of EC CoCos are uncertain at issuance, which can be specified as a form of expectation, $R_{EC} = E[S_{\tau_B}/C_p]$. In the following corollary, we formulate the expected recovery rates of EC CoCos (only when $w = 1$) at conversion with respect to the types of conversion prices in our framework. We assume that the initial RWA-per-share amount can be approximated from the CoCo market prices of each issuing bank.

Corollary 6 (Expected recovery rates of CoCos). Suppose that RWA-per-share L follows distribution F with initial value \bar{L} . Then, for fixed conversion price $C_p = S^*$, the expected recovery rate of CoCos with CET1 trigger α_0 is given by

$$R_{EC} = \frac{\alpha_0}{S^*} \bar{L}. \quad (8)$$

For floor conversion price $C_p = \max(S_{\tau_B}, S_F)$, the expected recovery rate of the CoCo is given by

$$R_{EC} = 1 - \frac{\alpha_0}{S_F} \int_0^{\frac{S_F}{\alpha_0}} F(x) dx. \quad (9)$$

As for a special case, we assume that the distribution of L is given as log-normal with standard deviation σ_L , i.e.

$$L = \bar{L} \exp\left(\sigma_L Z - \frac{1}{2}\sigma_L^2\right), \quad (10)$$

where Z is standard normal. Then, the expected recovery rate has the

following analytical form:

$$R_{EC} = \frac{\alpha_0}{S_F} \exp\left(\frac{1}{2}\Sigma^2 + m\right) \Phi(d_1) + \Phi(-d_2), \quad (11)$$

where m and Σ^2 are the corresponding normal mean and variance, respectively, i.e. $\ln(L) \sim \mathcal{N}(m, \Sigma^2)$, and

$$d_1 = \frac{1}{\Sigma} \left[\ln\left(\frac{S_F}{\alpha_0}\right) - (m + \Sigma^2) \right] \quad \text{and} \quad d_2 = d_1 + \Sigma.$$

Using a transformation formula between (m, Σ^2) and (\bar{L}, σ_L^2)

$$m = \ln(\bar{L}) + \frac{1}{2}\sigma_L^2 \quad \text{and} \quad \Sigma^2 = \sigma_L^2,$$

Eq. (11) can be expressed by the available parameters (\bar{L}, σ_L^2) at the evaluation date.⁴

4. Model for post-conversion risk

In this section, we propose a way to address the distinctive risk in EC CoCos. By assessing the propensity to issue CoCos with respect to the loss-absorbing methods (Fig. A.4) and CET1 trigger levels (Table A.2), we can investigate that (i) the proportion of EC CoCos issued gradually decreases, whereas that of WD CoCos increases⁵ and (ii) for the high-trigger (i.e. greater than or equal to 5%), EC CoCos account for 61.5% of the all CoCos issued; whereas for the low-trigger (i.e. less than 5%), WD CoCos account for 88.1%. This market share implies that the EC method is preferred for middle- and high-level trigger CoCos, whereas the WD method is preferred for low-trigger CoCos.

Suppose that all other conditions for EC and WD CoCos are the same including expected recovery rates. Then, we consider two situations: ex-ante and ex-post conversion. In ex-ante conversion, all the risk factors to which EC and WD CoCos are exposed are identical, since the expected recovery rates are the same. In ex-post conversion, however, different processes are activated in EC and WD CoCos. At conversion, EC CoCos turn into equities of the issuing bank, which can then be either liquidated or owned by shareholders. However, WD CoCos are terminated immediately at conversion. This fact indicates that some additional risk can be realized only in EC CoCos, not WD CoCos.

This additional risk primarily relies on the possibility that the issuing bank defaults or its equity is seriously impaired in the post-conversion period. Equity holders might be unable to sell their shares immediately at price S_{τ_B} because the conversion event can cause severe liquidity problems in the stock market. They may then be eventually exposed to the default risk of the CoCo-issuing bank if holding shares after conversion.

From this perspective, we define the *post-conversion risk* of CoCos as additional risk due to the likelihood of default (including a comprehensive default-like event such as the impairment or illiquidity of equity) of the issuing bank in the post-conversion period $(\tau_B, T]$.

4.1. Post-conversion risk model

When conversion occurs, the CET1 ratio of the CoCo issuer is updated under the pre-defined loss-absorbing method. The conversion of EC CoCos raises to increase the number of shares with the given conversion ratio, which automatically enhances the earlier CET1 ratio. To model the new CET1 ratio in the post-conversion period by using Eq. (2) we define the *adjusted CET1 ratio* as follows:

⁴ The expected value of L is equal to \bar{L} and the standard deviation of L is equal to $\bar{L} \sqrt{\exp(\sigma_L^2) - 1}$.

⁵ EC CoCos account for 46% and WD CoCos for 54% of the total market share of CoCos.

$$\text{Adjusted CET1 ratio} \approx \frac{S_t \times (M + C_r)}{\text{Total RWA}} = \frac{S_t}{\text{Total RWA}/(M + C_r)}, \quad (12)$$

where M is the number of existing shares before conversion and C_r is the conversion ratio. The conversion of CoCos does not change the amount of total RWA because CoCos are originally counted as either Additional Tier 1 or Tier 2 capital in calculation of RWA. Meanwhile, the conversion of WD CoCos does not improve the CET1 ratio as $C_r = 0$ in Eq. (12).

We denote L' as the value of each RWA-per-share diluted by EC, i.e.

$$L' = \frac{\text{Total RWA}}{(M + C_r)} = \frac{\text{Total RWA}/M}{(1 + C_r/M)}.$$

Here, L' is a non-negative random variable that depends on the variables of RWA-per-share L and conversion ratio C_r specified at issuance. Further, denote by

$$\psi(x) = \frac{x}{1 + C_r(x)/M}. \quad (13)$$

Then, $\psi(L)$ represents the diluted RWA-per-share variable in the post-conversion period. For the distribution F of L , the distribution of $L' = \psi(L)$,

$$Q(L' \leq y) = Q(\psi(L) \leq y) = F(\psi^{-1}(y)) \quad (14)$$

is well-defined since $\psi^{-1}(\cdot)$ always exists. The following lemma proves this.

Lemma 7. A real-valued function $\psi(x)$ defined as Eq. (13) is increasing, for all $x > 0$.

To build a model of post-conversion risk, we define another stopping time τ_D as the first passage time when the adjusted CET1 ratio reaches barrier α_1 after conversion τ_B ,

$$\tau_D = \inf \left\{ t \geq \tau_B: \frac{S_t}{L'} \leq \alpha_1 \right\} = \inf \{ t \geq \tau_B: S_t \leq \alpha_1 \psi(L) \}, \quad (15)$$

where α_1 is set below $\alpha_0 L / \psi(L)$, i.e. $\alpha_1 \psi(L) < \alpha_0 L$. $Q(\tau_B \leq \tau_D) = 1$ by definition. The barrier α_1 indicates the minimum threshold for the adjusted CET1 ratio required for which CoCo-issuing banks to continue operations (not default) in the post-conversion period under its regulatory environment. This can be interpreted as the minimum capital requirement that all banks must meet. For instance, α_1 can be set to 4.5% under Basel III for CET1 ratios.

In this context, we call stopping time τ_D the *regulatory default time*. This is different from the true default because the value of equity is zero when banks legally default or become insolvent. Upon regulatory default, however, banks may have a non-zero enterprise value although the CET1 ratio reaches regulatory threshold α_1 or equity price reaches $\alpha_1 \psi(L)$, as illustrated in Fig. 2.

The probability that a CoCo-issuing bank enters regulatory default after conversion τ_B is given in the following lemma.

Lemma 8. Suppose conversion occurs at time $t = \tau_B < T$. For $s \geq t$, the probability of regulatory default τ_D until time s , seen at time t is given by

$$Q(\tau_D \leq s | \mathcal{F}_t) = 1 - F \left(\psi^{-1} \left(\frac{m_t}{\alpha_1} \right) \right) + \int_0^{\psi^{-1} \left(\frac{m_t}{\alpha_1} \right)} H(s - t; \alpha_1 \psi(x), S_t) dF(x),$$

where F is the cumulative distribution of L before conversion and $m_t = \min_{0 \leq v \leq t} S_v$.

4.2. Quantifying post-conversion risk

Now, we propose a way to measure post-conversion risk by using the notion of regulatory default τ_D . We carry out the idea in barrier option pricing to assess the equity value at conversion τ_B . This approach suits the situation because if the issuing bank does not default until T , the final value of its equity would be S_T ; otherwise, the bank's equity

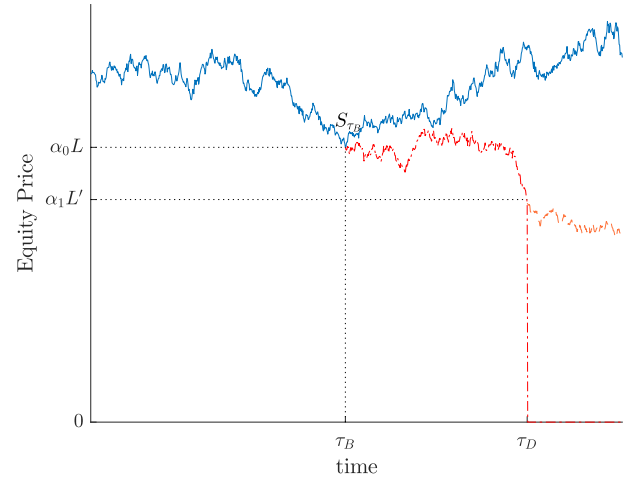


Fig. 2. Illustration of the sample paths of an equity price for regulatory default τ_D in ex-post conversion.

would plunge to zero upon default.

Remark 9 (Rubinstein & Reiner, 1991). (i) A down-and-out asset-or-nothing call option pays S_T if S_u does not fall below a fixed barrier $B < S_t$ at $t < u < T$; otherwise, pays zero. Then, the value of this option at t is given as

$$\begin{aligned} \zeta^{DO}(T-t, B, S_t) = & S_t \left[\Phi \left(\frac{\log \left(\frac{S_t}{B} \right) + \left(r + \frac{1}{2} \sigma^2 \right) (T-t)}{\sigma \sqrt{T-t}} \right) \right. \\ & \left. - \left(\frac{B}{S_t} \right)^{1+\frac{2r}{\sigma^2}} \Phi \left(\frac{-\log \left(\frac{S_t}{B} \right) + \left(r + \frac{1}{2} \sigma^2 \right) (T-t)}{\sigma \sqrt{T-t}} \right) \right]. \end{aligned} \quad (16)$$

(ii) A down-and-in asset-or-nothing call option pays S_T if S_u falls below a fixed barrier $B < S_t$ at $t < u < T$; otherwise pays zero. Then, the value of this option at t is given as

$$\begin{aligned} \zeta^{DI}(T-t, B, S_t) = & S_t \left[\Phi \left(\frac{-\log \left(\frac{S_t}{B} \right) - \left(r + \frac{1}{2} \sigma^2 \right) (T-t)}{\sigma \sqrt{T-t}} \right) \right. \\ & \left. + \left(\frac{B}{S_t} \right)^{1+\frac{2r}{\sigma^2}} \Phi \left(\frac{-\log \left(\frac{S_t}{B} \right) + \left(r + \frac{1}{2} \sigma^2 \right) (T-t)}{\sigma \sqrt{T-t}} \right) \right], \end{aligned} \quad (17)$$

where Φ is the standard normal distribution with risk-free rate r and stock price volatility σ .

To apply the possibility of regulatory default to CoCo's prices, we consider a type of virtual payoff reflecting the post-conversion risk that could occur after conversion - termed the 'hypothetical payoff' herein. Let ζ^{DO} be a barrier option price with strike $E = 0$ and random barrier $B = \alpha_1 L' = \alpha_1 \psi(L)$ in Eq. (16). The hypothetical payoff $K(\tau_B, S_{\tau_B})$ is defined as

$$K(\tau_B, S_{\tau_B}) = C_r(S_{\tau_B}) \zeta^{DO}(T - \tau_B, \alpha_1 \psi(L), S_{\tau_B}) + (1 - w) \delta N. \quad (18)$$

The meaning of the hypothetical payoff corresponds to the contractual payoff in CoCos replacing S_{τ_B} with $\zeta^{DO}(\cdot, S_{\tau_B})$ in Eq. (5). As barrier options are less worthy than the corresponding European options, we have

$$\bar{K}(\tau_B, S_{\tau_B}) - K(S_{\tau_B}) \geq 0, \quad (19)$$

where $\bar{K}(S_{\tau_B})$ is the contractual payoff in Eq. (5).

Based on the difference between the hypothetical and contractual

payoffs from an expectation perspective, we develop a formula for post-conversion risk amount $\mathcal{P}(\alpha_1)$.

$$\begin{aligned}\mathcal{P}(\alpha_1) &= E[e^{-r_B}(K(\tau_B, S_{\tau_B}) - \bar{K}(S_{\tau_B}))] \\ &= E\left[e^{-r_B}\left(\frac{S_{\tau_B}}{C_p} - \frac{\zeta^{DO}(S_{\tau_B}, \alpha_1\psi(L), T - \tau_B)}{C_p}\right)\right]\end{aligned}\quad (20)$$

$$= E\left[e^{-r_B}\frac{\zeta^{DI}(S_{\tau_B}, \alpha_1\psi(L), T - \tau_B)}{C_p}\right], \quad (21)$$

where ζ^{DI} is in Eq. (17). To obtain Eq. (21), we use $S_{\tau_B} = E[D(\tau_B, T)S_T | \mathcal{F}_{\tau_B}]$, which is equivalent to a regular European call with a zero strike. Then, we use the parity relation for knock-out and knock-in options⁶ since S_{τ_B} in Eq. (20) is the corresponding European call with a zero strike.

By calculating Eq. (21), we have

$$\begin{aligned}\mathcal{P}(\alpha_1) &= C_r(S_0) \int_{S_0/\alpha_0}^{\psi^{-1}(S_0/\alpha_1)} \zeta^{DI}(S_0, \alpha_1\psi(x), T) dF(x) \\ &\quad + \int_0^{S_0/\alpha_0} C_r(\alpha_0 x) \int_0^T \zeta^{DI}(\alpha_0 x, \alpha_1\psi(x), T \\ &\quad - s) D(0, s) h(s; \alpha_0 x, S_0) ds dF(x).\end{aligned}\quad (22)$$

The following theorem shows the zero-coupon CoCo that has the hypothetical payoff, i.e. reflects post-conversion risk until maturity T . **Theorem 10** (Zero-coupon CoCos with post-conversion risk). Suppose a zero-coupon CoCo with unit face value and maturity T . The zero-coupon CoCo value that reflects post-conversion risk on $[\tau_B, T]$ is given as

$$\begin{aligned}P^{ZC}(t, T) &= (1 - w)\delta \int_{\psi^{-1}(m_t/\alpha_1)}^{\infty} D(t, \tau_B^x) dF(x) \\ &\quad + \frac{(1 - w)\delta}{D(0, t)} \left(F\left(\psi^{-1}\left(\frac{m_t}{\alpha_1}\right)\right) - F\left(\frac{S_0}{\alpha_0}\right) \right) \\ &\quad + C_r(S_0) \int_{S_0/\alpha_0}^{\psi^{-1}(m_t/\alpha_1)} \zeta^{DO}(S_t, \alpha_1\psi(x), T - t) dF(x) \\ &\quad + \int_{m_t/\alpha_0}^{S_0/\alpha_0} K(\tau_B^x, \alpha_0 x) D(t, \tau_B^x) dF(x) \\ &\quad + \int_0^{m_t/\alpha_0} \int_t^T K(s, \alpha_0 x) D(t, s) h(s - t; \alpha_0 x, S_t) ds dF(x) \\ &\quad + D(t, T) \{1 - G_t(T)\},\end{aligned}\quad (23)$$

where K is the hypothetical payoff in Eq. (18).

We define the post-conversion risk premium $\mathcal{P}(\alpha_1)$ by computing the difference between the levels of yield-to-maturity (YTM) from two CoCo prices under the following two assumptions: (i) no post-conversion risk is reflected after conversion and (ii) post-conversion risk is reflected after conversion. The CoCo value under (i) and (ii) indicates a CoCo without post-conversion risk (Theorem 4 with a contractual payoff) and a CoCo with post-conversion risk (Theorem 10 with a hypothetical payoff), respectively.

For zero-coupon CoCos, we have

$$\mathcal{P}(\alpha_1) = -\frac{1}{T} \ln \frac{P^{ZC}(0, T)}{\bar{P}^{ZC}(0, T)}, \quad (24)$$

where \bar{P}^{ZC} and P^{ZC} are the T -maturity zero-coupon CoCo prices specified in Theorems 4 and 10, respectively. For coupon-bearing CoCos, we need to employ a numerical method (an iterative procedure, such as trial and error) to estimate the YTM from a bond price. Eq. (24) can be extended to general coupon-bearing types by computing the difference between the estimated YTMs of CoCos with/without post-conversion risk from the corresponding bond prices. As α_1 rises, the post-

Table 1

Empirical expected recovery rates R_{EC} evaluated at an issuance date for fixed conversion price EC CoCos undertaken by Barclays, HSBC, Lloyds, RBS, and SC (given the conversion ratio C_r , the fixed conversion price C_p , the stock price at issuance S_0 , the CET1 ratio trigger α_0).

CoCo issuer	ISIN	C_r	C_p	S_0	α_0	R_{EC}
Barclays	XS1002801758	605.7287	1.99	2.96	7.0%	29.7%
	US06738EAB11	605.3791	1.6519	2.18	7.0%	26.3%
	XS1068574828	740.5810	1.6515	2.18	7.0%	26.3%
	XS1068561098	741.2727	1.349	2.18	7.0%	32.3%
HSBC	XS1274156097	606.0606	1.65	2.78	7.0%	33.7%
	US404280AS86	229.5800	4.3558	6.40	7.0%	29.4%
	US404280AR04	229.5800	4.3558	6.40	7.0%	29.4%
	XS1111123987	296.2840	3.3751	6.40	7.0%	37.9%
Lloyds	XS0459090774	1535.3853	0.5921	0.55	5.0%	18.5%
	XS1043550307	1555.2100	0.6430	0.75	7.0%	23.4%
RBS	US539439AG42	932.8358	1.0720	1.29	7.0%	24.1%
	US780099CJ48	277.3156	3.606	3.44	7.0%	19.1%
SC	US780099CK11	277.3156	3.606	3.44	7.0%	19.1%
	US853254AT77	87.5350	11.424	10.3	7.0%	18.0%

conversion risk premium $\mathcal{P}(\alpha_1)$ increases. Fig. C6 illustrates that a CoCo price or its YTM changes depending on α_1 .

In this framework, a notable fact is that WD CoCos have a zero value of $\mathcal{P}(\alpha_1)$ because the probability of regulatory default is zero (WD contracts terminate at conversion). However, EC CoCos can have a positive value that depends on the level of regulatory default trigger α_1 since taking post-conversion risk. Thus, the proposed model enables us to differentiate between the methods of valuing EC and WD CoCos in that sense.

5. Empirical result

In this section, we assess the accuracy and efficiency of our theoretical formulae via a comparison with a full Monte Carlo method, and then carry out empirical tests based on the theoretical framework. For both the numerical and empirical tests, we assume that RWA-per-share level L follows a log-normal distribution⁷ in Eq. (10). Since the proposed formulae contain definite integral, a Gaussian quadrature scheme is used to estimate the integrations. Appendix C presents the results of the numerical simulations and the correctness of all proposed formulae can be confirmed by these tests. For the empirical tests, we obtain market price data from Thomson Reuters and use the CoCo issuance data collected by Moody's.

5.1. Estimation of the expected recovery rate of CoCos

We estimate the expected recovery rates evaluated at the issuance date by using Corollary 6 for the issued CoCos in practice. We select 14 EC CoCos with a fixed conversion price issued by major banks such as Barclays, HSBC, Lloyds Banking Group (Lloyds), Royal Bank of Scotland (RBS), and Standard Chartered (SC) before 3Q-2015. In this test, expected RWA level \bar{L} is assumed to be 20% of the stock price on the issue date because this is the average hidden stock trigger level (De Spiegeleer & Schoutens, 2013). Table 1 presents the empirical expected recovery rates of EC CoCos, which range from 18.0% to 37.9%.

5.2. Finding the post-conversion risk premium in the market

We examine whether the market prices of EC CoCos contain the post-conversion risk premium. We select the EC and WD CoCos of Credit Suisse, the only bank that issued both types of CoCos from 2009

⁶ For options with the same underlying, strike, and maturity, a regular European call (put) price is equal to the sum of prices of knock-out and knock-in calls (puts) that have the common barrier level.

⁷ For the simulation, we employ the setup used by the Technical Report, Finger, Finkelstein, Pan, Lardy, and Ta (2002) of the RiskMetrics Group, Chapter 2.

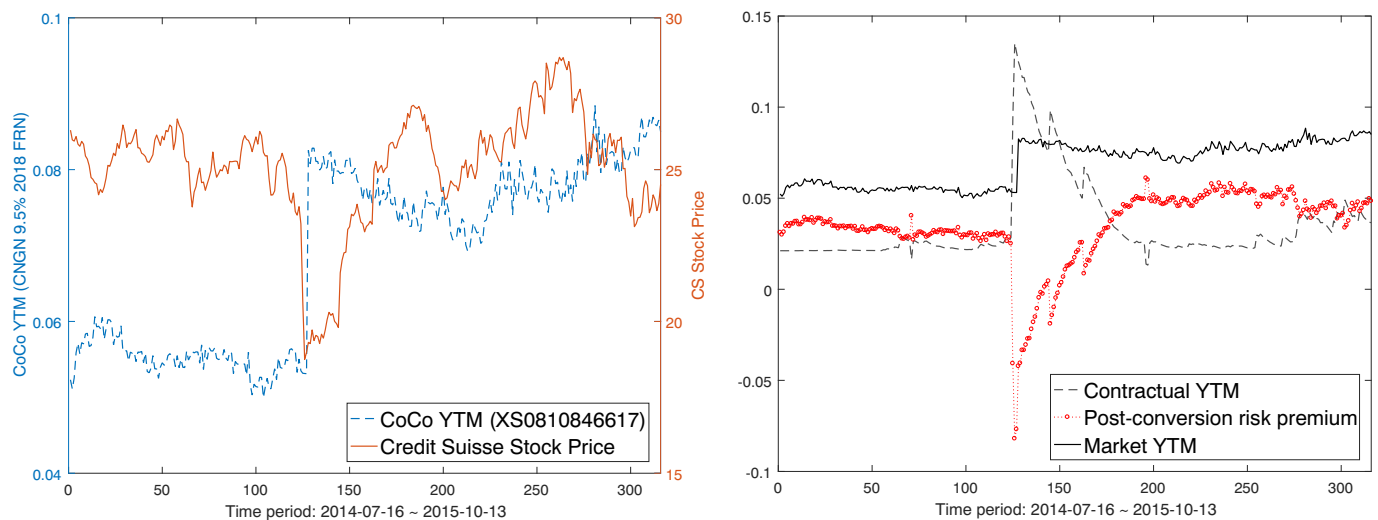


Fig. 3. Left: Market data of EC CoCos price and stock price of Credit Suisse. Right: Comparison of the theoretical values under the formula without post-conversion risk ('Contractual YTM'), the market value ('Market YTM'), and the empirical post-conversion risk premium of EC CoCos ('Post-conversion risk premium'). Sample period is 15 July 2014 to 13 October 2015.

to 3Q-2015. The test is conducted under the following three-step procedure:

- (i) Calibrate the model parameters from the WD CoCo market prices.
- (ii) Compute a theoretical price of the EC CoCo by using the formula for CoCos under the assumption that no post-conversion risk is assumed (Theorem 4) with the parameters calibrated from the WD CoCo in step (i).
- (iii) Compare the market price of the EC CoCo with the corresponding theoretical price under the assumption of no post-conversion risk, obtained in step (ii).

The samples used in this test are the WD CoCo (US22546DAB29) with a 7.5% semi-annual coupon, CET1 ratio trigger $\alpha_0 = 5.125\%$ issued on 11 December 2013; and the EC CoCo (XS0810846617) with a 9.5% semi-annual coupon, $\alpha_0 = 7\%$ issued on 21 July 2012.

In step (i), we estimate the most appropriate model parameters not directly observable in the market: mean \bar{L} and standard deviation σ_L under the log-normal assumption for a RWA-per-share variable L . By using the other observable information, \bar{L} , σ_L can be calibrated from a CoCo's market price P^{mrk} based on a least square method, i.e. $P(\cdot) = P^{\text{mrk}}$ with a corresponding theoretical price P . In this test, we assume $\sigma_L = 10\%$ which is the historical standard deviation of the rate of changes in the CET1 ratio of Credit Suisse. Then, we extract $\bar{L} = 146.8$ from the WD CoCo price.

In step (ii), we compute the theoretical value of the EC CoCo (XS0810846617) as a time series by using Theorem 4 with the parameters obtained in step (i)⁸. Here, it is reasonable to use the parameters from the WD CoCo to compute the EC CoCo because both CoCos were issued by the same bank. We assume that the parameters \bar{L} and σ_L from the WD CoCo price represent the capital-ratio of Credit Suisse. We conduct this test from 15 July 2014 to 13 October 2015 and compute the corresponding theoretical values for the EC CoCo (XS0810846617) assuming no post-conversion risk. For the test, we use the closing stock price of Credit Suisse Group AG (CGNG.VX) listed on the SIX⁹, and the EURIBOR swap rates for the risk-free rate. The left panel of Fig. 3

displays the time series for the market price of the EC CoCo (XS0810846617) and stock price of the Credit Suisse Group AG. The huge jumps in both time series on 15 January 2015 are due to a sudden announcement on its currency policy.¹⁰

In step (iii), we compare the market price with the theoretical values of the EC CoCo obtained in step (ii). We find that the market prices of the EC CoCo are constantly higher than the theoretical values, with an average difference of 2.00% in the test period. This result indicates that the market prices of the EC CoCo reflect a 2% post-conversion risk premium on average compared with the theoretical prediction without post-conversion risk. The right panel of Fig. 3 displays the time series of the market YTM of EC CoCos and their theoretical results under the assumption of no post-conversion risk.

As discussed by previous research that compares the different risk incentives in EC ad WD CoCos (Avdjiev et al., 2015; Martynova & Perotti, 2018), issuing EC CoCos can be more efficient than issuing the corresponding WD for issuers, but much riskier for investors, provided all other conditions are the same including the expected recovery rate. By assessing EC CoCos under the proposed framework, we find that they are traded in the market at a price including the 2% post-conversion risk premium on average. The result implies that such additional risk that only exists in EC CoCos can be quantified as 2% on average based on the market prices, which is consistent with the theory.

6. Concluding remarks

In this study, we propose a valuation framework for CET1 ratio trigger CoCos and validate the theoretical prediction empirically. First, we propose a new model for a CET1 ratio for pricing CoCos. We assume that a bank's equity value follows a geometric Brownian motion and that its RWA-per-share value is a single random variable that is unknown, where only the distribution is given. Under this setup, we obtain analytical formulae for the conversion probability, expected recovery rates, and CoCo values with a CET1 ratio trigger. Second, we quantify post-conversion risk that only exists in EC CoCos by finding the probability of regulatory default, which denotes that a CoCo-issuing bank defaults from a regulatory perspective after conversion (e.g. failing to meet the minimum capital requirements). Then, we derive the

⁸ The estimated \bar{L} corresponds to the expected stock barrier $E[S_{TB}] = 26.32\%$ for an initial stock price.

⁹ Credit Swiss Group AG is listed on the Swiss Exchange (SIX) and New York Stock Exchange.

¹⁰ Among many news articles, see <https://www.theguardian.com/commentisfree/2015/jan/16/switzerland-euro-crisis-swiss-franc-abolition-cap>.

post-conversion risk premium by computing the difference between the prices of CoCos with/without post-conversion risk.

In this framework, WD CoCos have a zero probability that regulatory default occurs after conversion, whereas EC CoCos have a positive probability in the post-conversion risk setting. We thus differentiate between the valuation methods for EC and WD CoCos in that sense. In practice, regardless of the loss-absorbing method is adopted, all CoCos are treated equally in terms of valuation, especially under regulation. Considering the pressure to recapitalize after the financial crisis and the favourable treatment of CoCos, a more rigorous analysis of EC and WD CoCos is necessary. Nonetheless, our study provides a more in-depth valuation of EC and WD CoCos than earlier studies.

Finally, our empirical analysis bridges the gap between our theoretical proposal and the market perception in terms of the loss-absorbing functions in CoCos. To examine the existence of the post-conversion risk premium in the real market, we conduct several tests by using the market data on CoCos issued by Credit Suisse. We find that market EC CoCo prices reflect a 2% post-conversion risk premium on average. This finding implies that issuers and investors of CoCos as well

as policymakers need to impose different risk charges or implement risk-taking incentives depending on the type of loss-absorbing method for CoCos.

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Appendix A. Figures and tables

All statistics data are based on a source by Moody's Quarterly Rated and Tracked CoCo Monitor Database during 4Q-2009 to 3Q-2015.

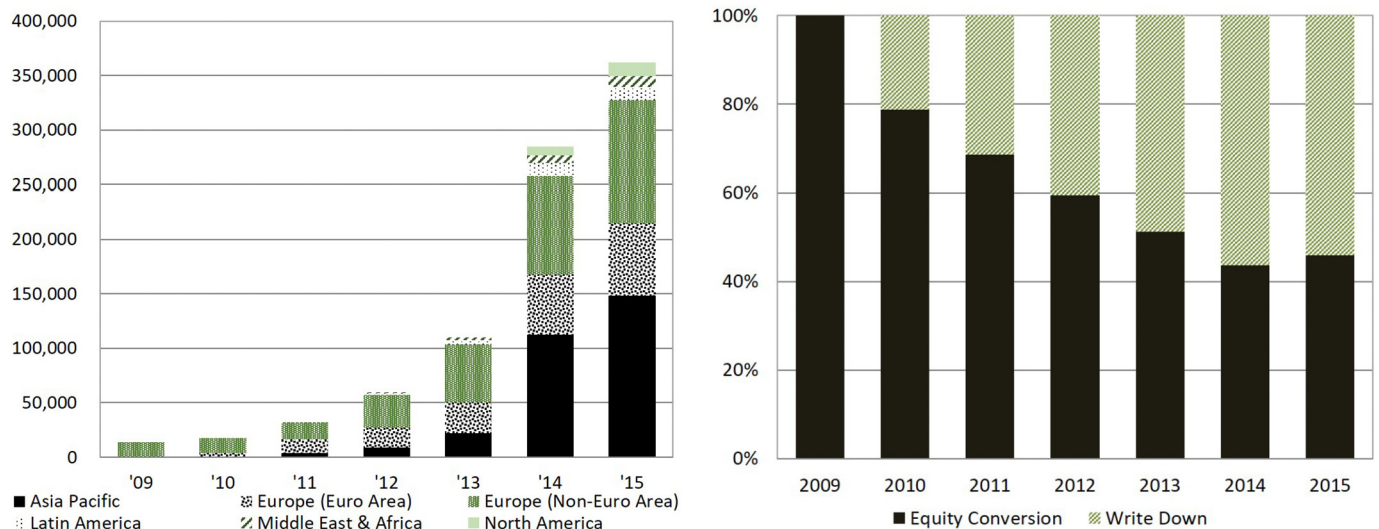


Fig. A.4. Left: Cumulative issuance amounts of CoCos with respect to a continental region. Right: proportion of cumulative issuance of CoCos with respect to loss-absorbing methods: equity-conversion and write-down. Sample period is 4Q-2009 to 3Q-2015.

Table A.2

Issuance proportion of CET1 trigger CoCos with respect to EC and WD conversion and with low-trigger, middle-trigger, and high-trigger levels. Sample period is 4Q-2009 to 3Q-2015.

CET1 trigger	Less than 5%	5% ≤ < 7%	Greater than 7%
EC CoCo	11.9%	61.3%	61.5%
WD CoCo	88.1%	38.7%	38.5%

Appendix B. Proofs

Proof of Lemma 7. Note that $C_r(x) = x/C_p(x)$ for C_p which is given in Eq. (4), i.e.,

$$C_p(x) = \max(\beta x, S^*)$$

and $\psi(x)$ equals

$$\psi(x) = \frac{x}{1 + \frac{x}{M \max(\beta x, S^*)}}.$$

For $\beta \neq 0$ and $x < \frac{S^*}{\beta}$, we have

$$\psi(x) = \frac{x}{1 + \frac{x}{MS^*}} = MS^* - \frac{MS^*}{1 + \frac{x}{MS^*}}, \quad (B.1)$$

thus ψ is increasing.

For $\beta \neq 0$ and $x \geq \frac{S^*}{\beta}$, we have

$$\psi(x) = \frac{x}{1 + \frac{1}{\beta M}},$$

and hence, ψ is an increasing function.

If $\beta = 0$, then Eq. (B.1) holds for all $0 < x < \infty$. Thus we obtain the desired result.

Proof of Lemma 3. For $s \geq t$, we obtain

$$\begin{aligned} G_t(s; x) &= \mathbb{P}(\tau_B^x \leq s | \mathcal{F}_t) = \mathbb{E}[1(\tau_B^x \leq s) | \mathcal{F}_t] \\ &= \mathbb{E}[1(\tau_B^x \leq t)1(\tau_B^x \leq s) | \mathcal{F}_t] + \mathbb{E}[1(\tau_B^x > t)1(\tau_B^x \leq s) | \mathcal{F}_t] \\ &= \mathbb{E}[1(\tau_B^x \leq t) | \mathcal{F}_t] + \mathbb{E}[1(t < \tau_B^x \leq s) | \mathcal{F}_t]. \end{aligned} \quad (B.2)$$

Since τ_B^x is a stopping time with respect to \mathcal{F}_t , so $\{\tau_B^x \leq t\}$ and $\{\tau_B^x > t\}$ are \mathcal{F}_t -measurable. Thus, Eq. (B.2) is equal to

$$1(\tau_B^x \leq t) + \mathbb{Q}(t < \tau_B^x \leq s | \mathcal{F}_t).$$

Since the events $\{t < \tau_B^x \leq s\}$ and $\{\tau_B^x > t\} \cap \{\min_{t < u \leq s} S_u \leq \alpha_0 x\}$ have the same probabilities for the \mathcal{F}_t -measurable set $\{\tau_B^x > t\}$, we have

$$\begin{aligned} \mathbb{Q}(t < \tau_B^x \leq s | \mathcal{F}_t) &= 1(\tau_B^x > t) \mathbb{Q}\left(\min_{t < u \leq s} S_u \leq \alpha_0 x | \mathcal{F}_t\right) \\ &= 1(\tau_B^x > t) \mathbb{Q}\left(\min_{t < u \leq s} \left[S_t e^{\left(r - \frac{1}{2}\sigma^2\right)(u-t) + \sigma(W_u - W_t)}\right] \leq \alpha_0 x \mid \mathcal{F}_t\right) \\ &= 1(\tau_B^x > t) \mathbb{Q}\left(\min_{t < u \leq s} \left[\left(r - \frac{1}{2}\sigma^2\right)(u - t) + \sigma(W_u - W_t)\right] \leq \ln\left(\frac{\alpha_0 x}{S_t}\right) \mid \mathcal{F}_t\right) \\ &= 1(\tau_B^x > t) H(s - t; \alpha_0 x, S_t). \end{aligned}$$

According to Remark 1, the distribution of the first hitting time of the geometric Brownian motion is given by $H(s - t; \alpha_0 x, S_t)$ with $B = \alpha_0 x$ and an initial point S_t during the time period $s - t$.

Let $G_t(s)$ be the conditional distribution of τ_B for any time $s \geq t$, given the observation up to time t , that is, $\mathbb{Q}(\tau_B \leq s | \mathcal{F}_t)$ with a filtration $\mathcal{F}_t = \sigma(S_u; 0 \leq u \leq t)$. Thus, for $m_t = \min_{0 \leq v \leq t} S_v$, we have

$$\begin{aligned} G_t(s) &= \mathbb{P}(\tau_B \leq s | \mathcal{F}_t) = \mathbb{P}(\tau_B \leq t | \mathcal{F}_t) + \mathbb{P}(t < \tau_B \leq s | \mathcal{F}_t) \\ &= 1 - F\left(\frac{m_t}{\alpha_0}\right) + \mathbb{E}[1(t < \tau_B \leq s) | \mathcal{F}_t]. \end{aligned}$$

Hence, Lemma 3 implies as follows:

$$\begin{aligned} \mathbb{E}[1(t < \tau_B \leq s) | \mathcal{F}_t] &= \int_0^\infty \mathbb{E}[1(t < \tau_B^x \leq s) | \mathcal{F}_t] dF(x) \\ &= \int_0^\infty 1(\tau_B^x > t) H(s - t; \alpha_0 x, S_t) dF(x) \\ &= \int_0^{m_t} H(s - t; \alpha_0 x, S_t) dF(x), \end{aligned}$$

where τ_B^x is defined in Definition 2.

Proof of Theorem 4. The discounted cash flows of a CoCo that pays contractual payoff at time t is written by

$$1(\tau_B \leq T) \bar{K}(S_{\tau_B}) D(t, \tau_B) + 1(\tau_B > T) D(t, T), \quad (B.3)$$

where $\bar{K}(S_{\tau_B})$ is the contractual payoff in Eq. (5) and $1(\cdot)$ is a characteristic function.

$$\begin{aligned} C(t, T) &= \mathbb{E}[1(\tau_B \leq T) \bar{K}(S_{\tau_B}) D(t, \tau_B) + 1(\tau_B > T) D(t, T) | \mathcal{F}_t] \\ &= \mathbb{E}[1(\tau_B \leq T) \bar{K}(S_{\tau_B}) D(t, \tau_B) | \mathcal{F}_t] + \mathbb{E}[1(\tau_B > T) D(t, T) | \mathcal{F}_t]. \end{aligned} \quad (B.4)$$

The first term of Eq. (B.4) is equivalent with as follows: for $t \leq T$,

$$\begin{aligned} &\mathbb{E}[1(\tau_B \leq T) \bar{K}(S_{\tau_B}) D(t, \tau_B) | \mathcal{F}_t] \\ &= \int_0^\infty \mathbb{E}[1(\tau_B^x \leq T) \bar{K}(S_{\tau_B^x}) D(t, \tau_B^x) | \mathcal{F}_t] dF(x) \\ &= \int_0^\infty \mathbb{E}[1(\tau_B^x \leq t) \bar{K}(S_{\tau_B^x}) D(t, \tau_B^x) | \mathcal{F}_t] dF(x) \end{aligned} \quad (B.5)$$

$$+ \int_0^\infty \mathbb{E}[1(\tau_B^x > t) 1(\tau_B^x \leq T) \bar{K}(S_{\tau_B^x}) D(t, \tau_B^x) | \mathcal{F}_t] dF(x). \quad (B.6)$$

In case when conversion occurs between time 0 and t , an equity price S_{τ_B} at the moment of conversion is equal to $\alpha_0 x$. Since the set $\{\tau_B^x \leq t\}$ is equivalent to the set $\{m_t \leq \alpha_0 x\}$ up to time t , Eq. (B.5) is derived as follows:

$$\begin{aligned} & \int_0^\infty \mathbb{E}[1(\tau_B^x \leq t) \bar{K}(S_{\tau_B^x}) D(t, \tau_B^x) | \mathcal{F}_t] dF(x) \\ &= \int_0^\infty 1(m_t \leq \alpha_0 x) \bar{K}(S_{\tau_B^x}) D(t, \tau_B^x) dF(x) \\ &= \int_{\frac{m_t}{\alpha_0}}^{\frac{S_0}{\alpha_0}} \bar{K}(S_{\tau_B^x}) D(t, \tau_B^x) dF(x) + \int_{\frac{S_0}{\alpha_0}}^\infty \bar{K}(S_{\tau_B^x}) D(t, \tau_B^x) dF(x) \\ &= \int_{\frac{m_t}{\alpha_0}}^{\frac{S_0}{\alpha_0}} \bar{K}(\alpha_0 x) D(t, \tau_B^x) dF(x) + \int_{\frac{S_0}{\alpha_0}}^\infty \bar{K}(S_0) \frac{1}{D(0, t)} dF(x) \\ &= \int_{\frac{m_t}{\alpha_0}}^{\frac{S_t}{\alpha_0}} \bar{K}(\alpha_0 x) D(t, \tau_B^x) dF(x) + \frac{\bar{K}(S_0)}{D(0, t)} \left\{ 1 - F\left(\frac{S_0}{\alpha_0}\right) \right\}. \end{aligned}$$

In case when conversion occurs after time t , using Lemma 1, Eq. (B.6) is given as follows:

$$\begin{aligned} & \int_0^\infty \mathbb{E}[1(\tau_B^x > t) 1(\tau_B^x \leq T) \bar{K}(S_{\tau_B^x}) D(t, \tau_B^x) | \mathcal{F}_t] dF(x) \\ &= \int_0^\infty 1(\tau_B^x > t) \bar{K}(\alpha_0 x) \mathbb{E}[1(\tau_B^x \leq T) D(t, \tau_B^x) | \mathcal{F}_t] dF(x) \\ &= \int_0^\infty 1(\tau_B^x > t) \bar{K}(\alpha_0 x) \int_t^T D(t, x) \frac{d}{ds} \mathbb{P}(\tau_B^x \leq s | \mathcal{F}_t) ds dF(x) \\ &= \int_0^\infty 1(\tau_B^x > t) \bar{K}(\alpha_0 x) \int_t^T D(t, s) 1(\tau_B^x > t) h(s - t; \alpha_0 x, S_t) ds dF(x) \\ &= \int_0^{\frac{m_t}{\alpha_0}} \bar{K}(\alpha_0 x) \int_t^T D(t, s) h(s - t; \alpha_0 x, S_t) ds dF(x) \\ &= \int_0^{\frac{m_t}{\alpha_0}} \bar{K}(\alpha_0 x) \tilde{H}(T - t; \alpha_0 x, S_t) dF(x). \end{aligned}$$

Furthermore, the second term of Eq. (B.4) is written as follows:

$$\begin{aligned} \mathbb{E}[1(\tau_B > T) D(t, T) | \mathcal{F}_t] &= D(t, T) \mathbb{E}[1(\tau_B > T) | \mathcal{F}_t] \\ &= D(t, T) \{1 - G_t(T)\}. \end{aligned}$$

Proof of Theorem 10. The discounted cash flows of a CoCo that pays hypothetical payoff at time t is written by

$$1(\tau_B \leq T) K(\tau_B, S_{\tau_B}) D(t, \tau_B) + 1(\tau_B > T) N D(t, T), \quad (\text{B.7})$$

where $K(S_{\tau_B})$ is the hypothetical payoff in Eq. (18).

$$\begin{aligned} P^{ZC}(t, T) &= \mathbb{E}[1(\tau_B \leq T) K(\tau_B, S_{\tau_B}) D(t, \tau_B) + 1(\tau_B > T) N D(t, T) | \mathcal{F}_t] \\ &= \mathbb{E}[1(\tau_B \leq T) K(\tau_B, S_{\tau_B}) D(t, \tau_B) | \mathcal{F}_t] + \mathbb{E}[1(\tau_B > T) N D(t, T) | \mathcal{F}_t]. \end{aligned} \quad (\text{B.8})$$

The first term of Eq. (B.8) is equivalent with as follows: for $t \leq T$,

$$\begin{aligned} & \mathbb{E}[1(\tau_B \leq T) K(\tau_B, S_{\tau_B}) D(t, \tau_B) | \mathcal{F}_t] \\ &= \int_0^\infty \mathbb{E}[1(\tau_B^x \leq T) K(\tau_B^x, S_{\tau_B^x}) D(t, \tau_B^x) | \mathcal{F}_t] dF(x) \\ &= \int_0^\infty \mathbb{E}[1(\tau_B^x \leq t) K(\tau_B^x, S_{\tau_B^x}) D(t, \tau_B^x) | \mathcal{F}_t] dF(x) \\ &+ \int_0^\infty \mathbb{E}[1(\tau_B^x > t) 1(\tau_B^x \leq T) K(\tau_B^x, S_{\tau_B^x}) D(t, \tau_B^x) | \mathcal{F}_t] dF(x). \\ &= \int_0^\infty \mathbb{E}[1(\tau_B^x \leq t) 1(\tau_D^x \leq t) K(\tau_B^x, S_{\tau_B^x}) D(t, \tau_B^x) | \mathcal{F}_t] dF(x) \end{aligned} \quad (\text{B.9})$$

$$+ \int_0^\infty \mathbb{E}[1(\tau_B^x \leq t) 1(\tau_D^x > t) K(\tau_B^x, S_{\tau_B^x}) D(t, \tau_B^x) | \mathcal{F}_t] dF(x) \quad (\text{B.10})$$

$$+ \int_0^\infty \mathbb{E}[1(\tau_B^x > t) 1(\tau_B^x \leq T) K(\tau_B^x, S_{\tau_B^x}) D(t, \tau_B^x) | \mathcal{F}_t] dF(x). \quad (\text{B.11})$$

In case when both conversion and regulatory default occur before t , the payoff $(1 - w)\delta$ was paid at τ_B . Thus, as the set $\{\tau_D \leq t\}$ is equivalent to the set $m_t \leq \alpha_1 \psi(x)$ up to time t , Eq. (B.9) implies

$$\int_0^\infty \mathbb{E}[1(\tau_B^x \leq t)1(\tau_D^x \leq t)K(\tau_B^x, S_{\tau_B^x})D(t, \tau_B^x)|\mathcal{F}_t]dF(x) = (1 - w)\delta \int_{\psi^{-1}(m_t/\alpha_1)}^\infty D(t, \tau_B^x)dF(x).$$

For Eq. (B.10), we have

$$\begin{aligned} & \int_0^\infty \mathbb{E}[1(\tau_B^x \leq t)1(\tau_D^x > t)K(\tau_B^x, S_{\tau_B^x})D(t, \tau_B^x)|\mathcal{F}_t]dF(x) \\ &= \int_{m_t/\alpha_0}^{\psi^{-1}(m_t/\alpha_1)} K(\tau_B^x, S_{\tau_B^x})D(t, \tau_B^x)dF(x) \\ &= \int_{S_0/\alpha_0}^{\psi^{-1}(m_t/\alpha_1)} K(\tau_B^x, S_{\tau_B^x})D(t, \tau_B^x)dF(x) + \int_{m_t/\alpha_0}^{S_0/\alpha_0} K(\tau_B^x, S_{\tau_B^x})D(t, \tau_B^x)dF(x) \\ &= \int_{S_0/\alpha_0}^{\psi^{-1}(m_t/\alpha_1)} \left(C_r(S_0)\zeta(T - t, \alpha_1\psi(x), S_t) + (1 - w)\delta \frac{1}{D(0, t)} \right) dF(x) \\ &+ \int_{m_t/\alpha_0}^{S_0/\alpha_0} K(\tau_B^x, \alpha_0 x)D(t, \tau_B^x)dF(x). \end{aligned} \tag{B.12}$$

Eq. (B.12) implies

$$\begin{aligned} & \frac{(1 - w)\delta}{D(0, t)} \left(F\left(\psi^{-1}\left(\frac{m_t}{\alpha_1}\right)\right) - F\left(\frac{S_0}{\alpha_0}\right) \right) + C_r(S_0) \int_{S_0/\alpha_0}^{\psi^{-1}(m_t/\alpha_1)} \zeta(T - t, \alpha_1\psi(x), S_t)dF(x) \\ &+ \int_{m_t/\alpha_0}^{S_0/\alpha_0} K(\tau_B^x, \alpha_0 x)D(t, \tau_B^x)dF(x). \end{aligned}$$

Here we have used the fact that an equity price at conversion $S_{\tau_B^x}$ is equal to S_0 on $\{x : S_0 \leq \alpha_0 x\}$.

Similar to the proof of Theorem 4, in case when conversion occurs after time t , using Lemma 1, Eq. (B.11) is given as follows:

$$\begin{aligned} & \int_0^\infty \mathbb{E}[1(\tau_B^x > t)1(\tau_D^x \leq T)K(\tau_B^x, S_{\tau_B^x})D(t, \tau_B^x)|\mathcal{F}_t]dF(x) \\ &= \int_0^\infty 1(\tau_B^x > t)\mathbb{E}[1(\tau_B^x \leq T)K(\tau_B^x, S_{\tau_B^x})D(t, \tau_B^x)|\mathcal{F}_t]dF(x) \\ &= \int_0^\infty 1(\tau_B^x > t) \int_t^T K(s, \alpha_0 x)D(t, x) \frac{d}{ds} Q(\tau_B^x \leq s | \mathcal{F}_t) ds dF(x) \\ &= \int_0^\infty 1(\tau_B^x > t) \int_t^T K(s, \alpha_0 x)D(t, s)1(\tau_B^x > t)h(s - t; \alpha_0 x, S_t) ds dF(x) \\ &= \int_0^{m_t/\alpha_0} \int_t^T K(s, \alpha_0 x)D(t, s)h(s - t; \alpha_0 x, S_t) ds dF(x). \end{aligned}$$

Furthermore, the second term of Eq. (B.8) is written as follows:

$$\begin{aligned} \mathbb{E}[1(\tau_B > T)D(t, T)|\mathcal{F}_t] &= D(t, T)\mathbb{E}[1(\tau_B > T)|\mathcal{F}_t] \\ &= D(t, T)\{1 - G_t(T)\}. \end{aligned}$$

Proof of Corollary 6. For Eq. (8), by definition of a fixed conversion price, we have

$$R_{EC} = E\left[\frac{S_{\tau_B}}{S^*}\right] = \frac{\alpha_0}{S^*} E[L] = \frac{\alpha_0}{S^*} \bar{L}.$$

For Eq. (9), by definition of a floor conversion price, we have

$$\begin{aligned} R_{EC} &= E\left[\frac{S_{\tau_B}}{\max(S_{\tau_B}, S_F)}\right] = E\left[\frac{\alpha_0 L}{S_F} \cdot 1\left(L \leq \frac{S_F}{\alpha_0}\right) + 1\left(L > \frac{S_F}{\alpha_0}\right)\right] \\ &= \frac{\alpha_0}{S_F} \int_0^{\frac{S_F}{\alpha_0}} x dF(x) + 1 - F\left(\frac{S_F}{\alpha_0}\right) = 1 - \frac{\alpha_0}{S_F} \int_0^{\frac{S_F}{\alpha_0}} F(x) dx. \end{aligned}$$

Note that integration-by-parts is used to hold the last equality in Eq. (9).

For Eq. (11), we assume that F is a cumulative distribution of a log-normal variable L and its logarithm has mean m and standard deviation Σ . We have to compute $F(A)$ and $\int_0^A x dF(x)$ for $A > 0$ to obtain the recovery rate with floor conversion price in Eq. (9)

$$F(A) = \frac{1}{x\sqrt{2\pi}} \int_0^A \frac{1}{x} \exp\left(-\frac{(\ln x - m)^2}{2\Sigma^2}\right) dx.$$

Let $t = \ln x$, then we have

$$\frac{1}{\Sigma\sqrt{2\pi}} \int_{-\infty}^{\ln A} \exp\left(-\frac{(t - m)^2}{2\Sigma^2}\right) dt.$$

Substituting again $y = \frac{t - m}{\Sigma}$, we have

$$F(A) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\ln A - m}{\Sigma}} e^{-\frac{1}{2}y^2} dy = \Phi\left(\frac{\ln A - m}{\Sigma}\right). \quad (\text{B.13})$$

Next,

$$\int_0^A x dF(x) = \frac{1}{\Sigma\sqrt{2\pi}} \int_0^A \exp\left(-\frac{(\ln x - m)^2}{2\Sigma^2}\right) dx.$$

Let $t = \ln x$, then we have

$$\begin{aligned} \int_0^A x dF(x) &= \frac{1}{\Sigma\sqrt{2\pi}} \int_{-\infty}^{\ln A} \exp\left(-\frac{(t - m)^2}{2\Sigma^2} + t\right) \\ &= \frac{1}{\Sigma\sqrt{2\pi}} \int_{-\infty}^{\ln A} \exp\left(-\frac{(t - (\Sigma^2 + m))^2}{2\Sigma^2}\right) dt \times \exp\left(\frac{1}{2}\Sigma^2 + m\right). \end{aligned}$$

Let $y = \frac{(t - (\Sigma^2 + m))}{\Sigma}$, we have

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{\ln A - (\Sigma^2 + m)}{\Sigma}} e^{-\frac{1}{2}y^2} dy \times e^{\frac{1}{2}\Sigma^2 + m} = \Phi\left(\frac{\ln A - (\Sigma^2 + m)}{\Sigma}\right) \times e^{\frac{1}{2}\Sigma^2 + m}. \quad (\text{B.14})$$

By Eqs. (B.13) and (B.14), the expected recovery rate with the floor conversion price is given as

$$R_{EC} = 1 + \frac{\alpha_0}{S^*} \Phi\left(\frac{\ln\left(\frac{S^*}{\alpha_0}\right) - (\Sigma^2 + m)}{\Sigma}\right) e^{\frac{1}{2}\Sigma^2 + m} - \Phi\left(\frac{\ln\left(\frac{S^*}{\alpha_0}\right) - m}{\Sigma}\right).$$

B.1. Application the proposed model to a fixed-income derivatives approach

De Spiegeleer and Schoutens (2010, 2012) propose the derivatives-based model. Here, CoCos are considered to have both properties, equity and fixed-income derivatives. In the equity derivatives approach, a CoCo is divided into three building blocks: a zero-coupon bond without a conversion feature, a knock-in forward, and a down-and-in digital barrier option on the shares of an issuing bank. These three components can be computed under the Black-Scholes framework. The hidden equity barrier in knock-in options is estimated from market CoCo prices. This approach was mentioned in the main text.

In the fixed-income approach, CoCo spreads are denoted as an extra return over the risk-free rate and they can be derived in a similar way to computing CDS spread. To find the conversion probability, conversion intensity is estimated from the market spreads of CoCos, which are higher than the default intensity of an issuing bank. Based on the credit triangle for CDS spreads, a similar relation can be derived among a CoCo spread, its recovery rate, and conversion probability (intensity). Denote $s(T)$ by a T -maturity CoCo spread for a T -maturity CoCo. Then, the estimate of $s(T)$ is constructed as

$$s(T) = (1 - R) \times \bar{\lambda}_T, \quad (\text{B.15})$$

where R is the expected recovery rate derived as a formula in Corollary 6 and $\bar{\lambda}_T$ is conversion intensity, which is calculated by taking the average of instantaneous conversion intensities $\lambda(t)$ over $[0, T]$ such that

$$\bar{\lambda}_T = \frac{1}{T} \int_0^T \lambda(t) dt, \quad \text{where } \lambda(t) = \lim_{\Delta t \rightarrow 0} \frac{1}{\Delta t} \mathbb{P}(\tau_B \leq t + \Delta t | \tau_B > t).$$

Under our framework, the intensity of conversion $\lambda(t)$ is estimated as

$$\lambda(t) \approx \frac{1}{\Delta t} \frac{G_0(t + \Delta t) - G_0(t)}{1 - G_0(t)} = \frac{\int_0^{\frac{S_0}{\alpha_0}} h(t; \alpha_0 x, S_0) dF(x)}{F\left(\frac{S_0}{\alpha_0}\right) - \int_0^{\frac{S_0}{\alpha_0}} H(t; \alpha_0 x, S_0) dF(x)},$$

where $G_0(t)$ is the conversion probability up to time t as seen from $t = 0$.

Appendix C. Numerical tests in Section 5

For the numerical tests, the following are chosen: an EC CoCo with a 6% coupon provided semi-annually, stock price $S_0 = 100$, risk-free interest rate $r = 3\%$, stock price volatility $\sigma = 20\%$, expected RWA-per-share level $\bar{L} = 500$, fixed conversion price $C_p = 2S_0$, floor conversion price $C_p = \max(S_{\tau_B}, 60)$, maturity $T = 20$ years, conversion trigger level $\alpha_0 = 5\%$, post-conversion default (regulatory default) level $\alpha_1 = 3\%$, face value $N = 100$, and EC proportion $w = 1$.

C.1. Probability distributions of conversion time

Lemma 3 shows the distribution of conversion time τ_B in Eq. (7). Fig. C1 shows the cumulative distribution $G_0(t)$ of conversion time τ_B with different choices of conversion trigger levels $\alpha_0 = 10\%, 7\%, 5\%$ over time $0 \leq t \leq 50$ years. As conversion trigger level α_0 falls, the probability of conversion decreases. As usual, conversion trigger levels are set between 5% and 8% in practice.

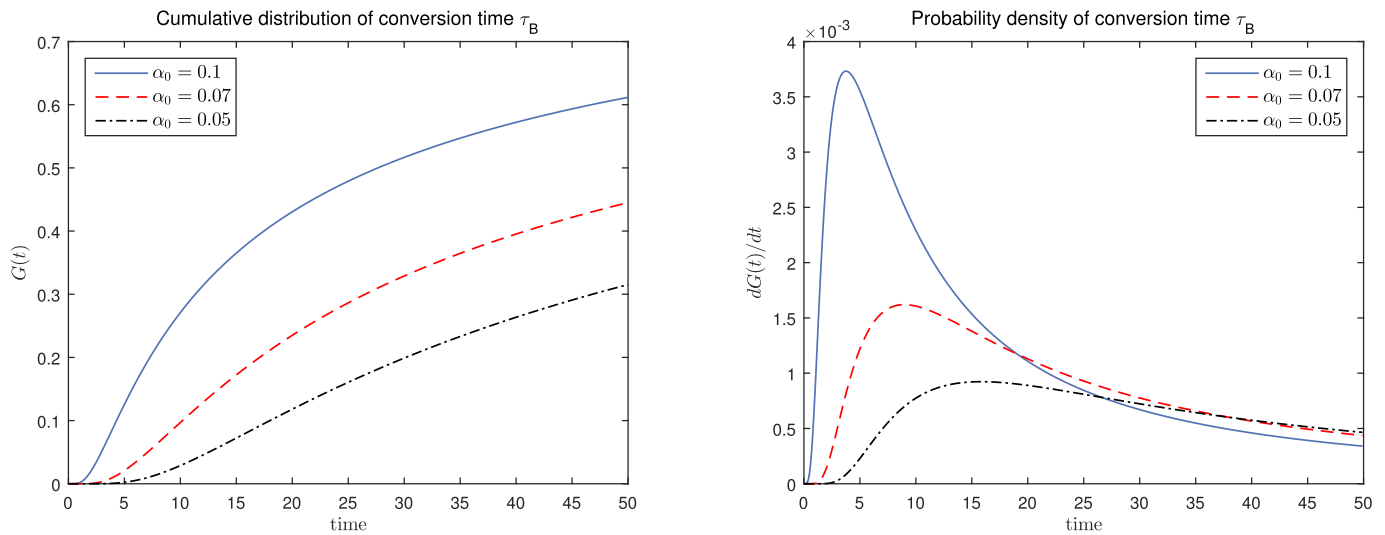


Fig. C1. A cumulative distribution (left) and a probability density function (right) for conversion time when trigger levels $\alpha_0 = 10\%, 7\%, 5\%$.

C.2. Comparison with full Monte Carlo methods

Figs. C2 and C3 display the CoCo prices at $t = 0$ without post-conversion risk (Theorem 4) and with post-conversion risk (Theorem 10) depending on maturity 5 to 50 years, respectively. The left panels show the results from a Monte Carlo method and its statistical estimate given by 99% confidence intervals with an I-shaped bar. In this simulation, fixed conversion price $C_p = S_0$ is used. We confirm that the values estimated by using our method are in the confidence interval.

In Figs. C2 and C3, the right panels show the running time to implement the numerical results by the analytic formula and a Monte Carlo method. The computing time remains similar when using the analytic formula, while that of the Monte Carlo method increases approximately linearly as the maturity increases.

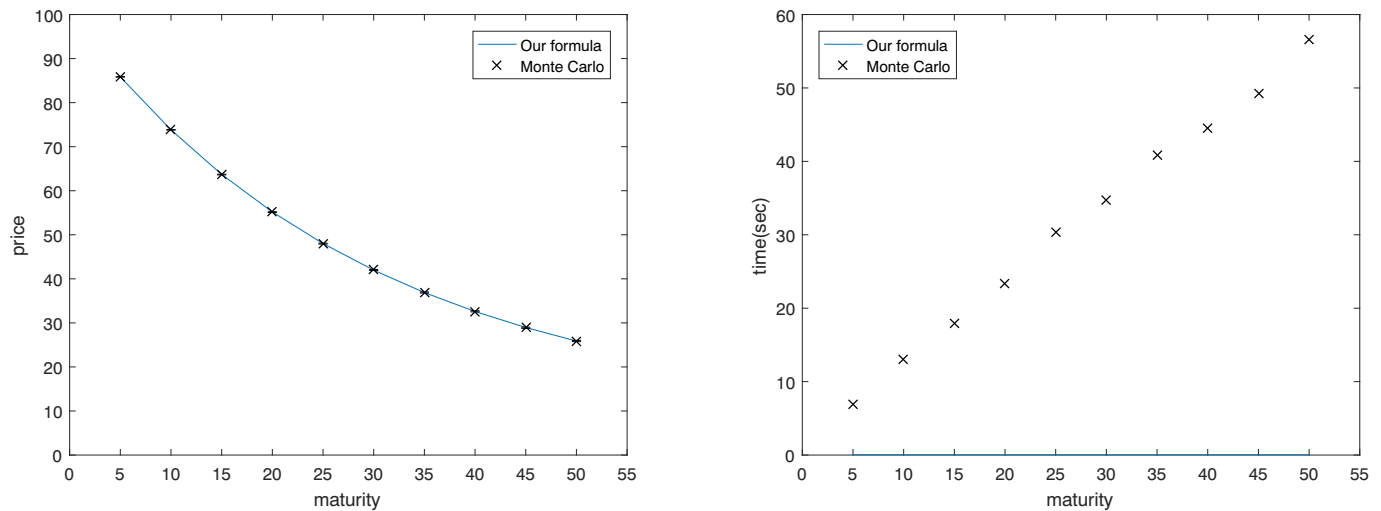


Fig. C2. Comparison of the CoCo prices without post-conversion risk (Theorem 4) obtained by the analytic formula and a Monte Carlo method with respect to time-to-maturity (left) and the corresponding computation time (right).

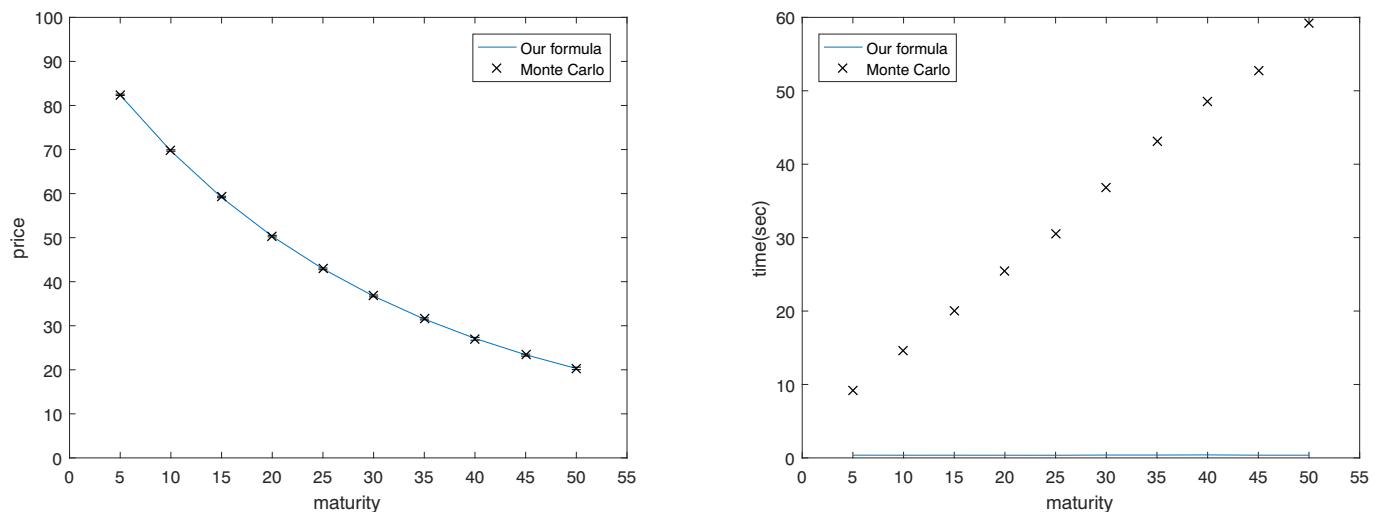


Fig. C3. Comparison of the CoCo prices with post-conversion risk (Theorem 10) obtained by the analytic formula and a Monte Carlo method with respect to time-to-maturity (left) and the corresponding computation time (right).

C.3. CoCo prices with/without post-conversion risk

The tests show sensitivity for a zero-coupon EC CoCo price and its YTM with/without post-conversion risk when the maturity changes from 1 to 40 years at $S_0 = 100$, and the stock price from 40 to 180 at $T = 10$. Figs. C4 and C5 display the results with the fixed/floor conversion price, respectively. In this simulation, we use risk-free rate $r = 2.1\%$, initial RWA-per-share level $\bar{L} = 700$, stock floor level $S_F = 70$, conversion trigger $\alpha_0 = 5.125\%$, and regulatory default trigger $\alpha_1 = 4.5\%$. The EC CoCo prices with post-conversion risk are less than those without post-conversion risk, while YTM of EC CoCos with post-conversion risk are greater than those without post-conversion risk for all choices of maturity and stock prices.

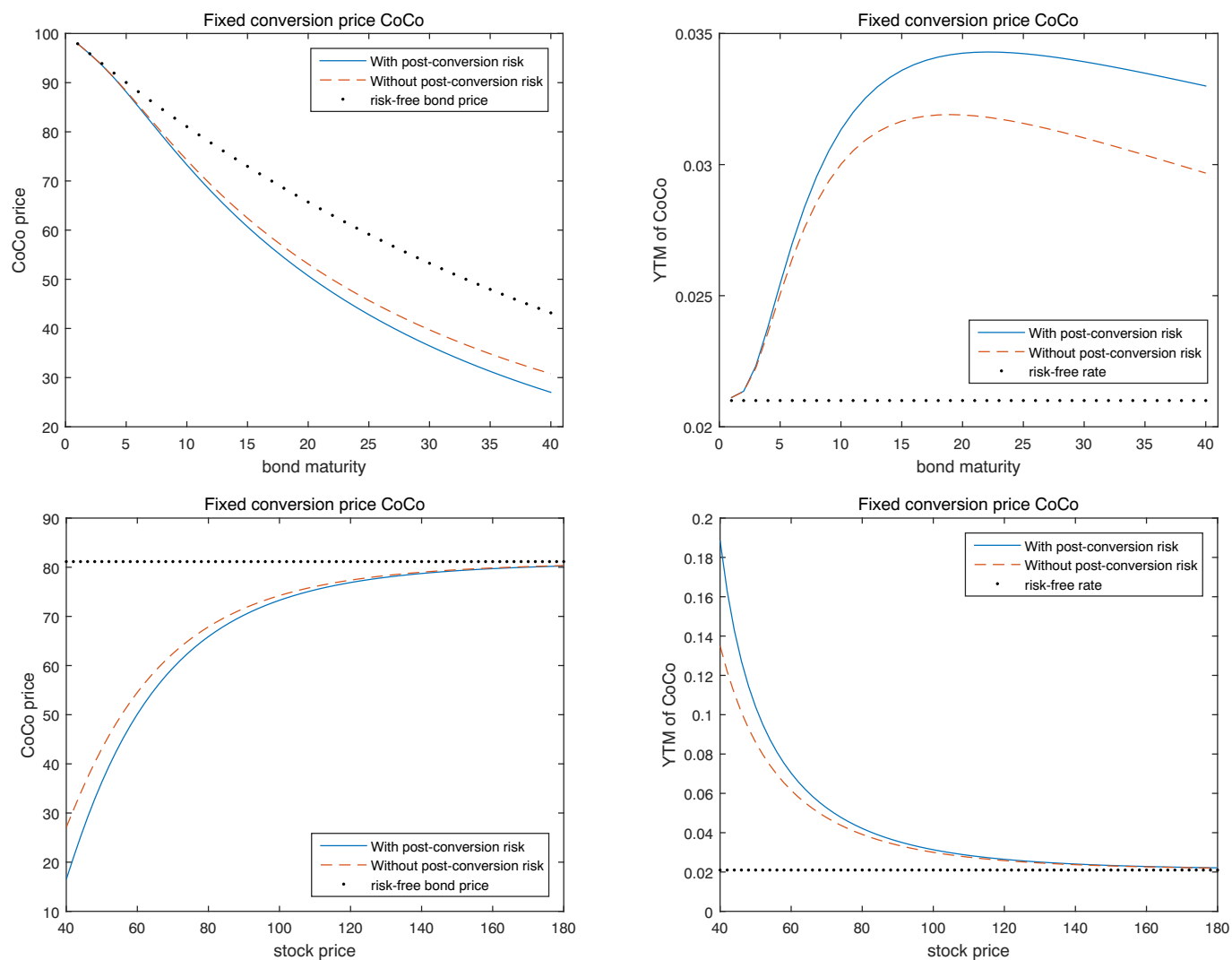


Fig. C4. Comparison of CoCo prices (left) and their YTM (right) for fixed conversion prices with and without post-conversion risk with a baseline of a risk-free bond price against maturities (top) and equity prices (bottom).

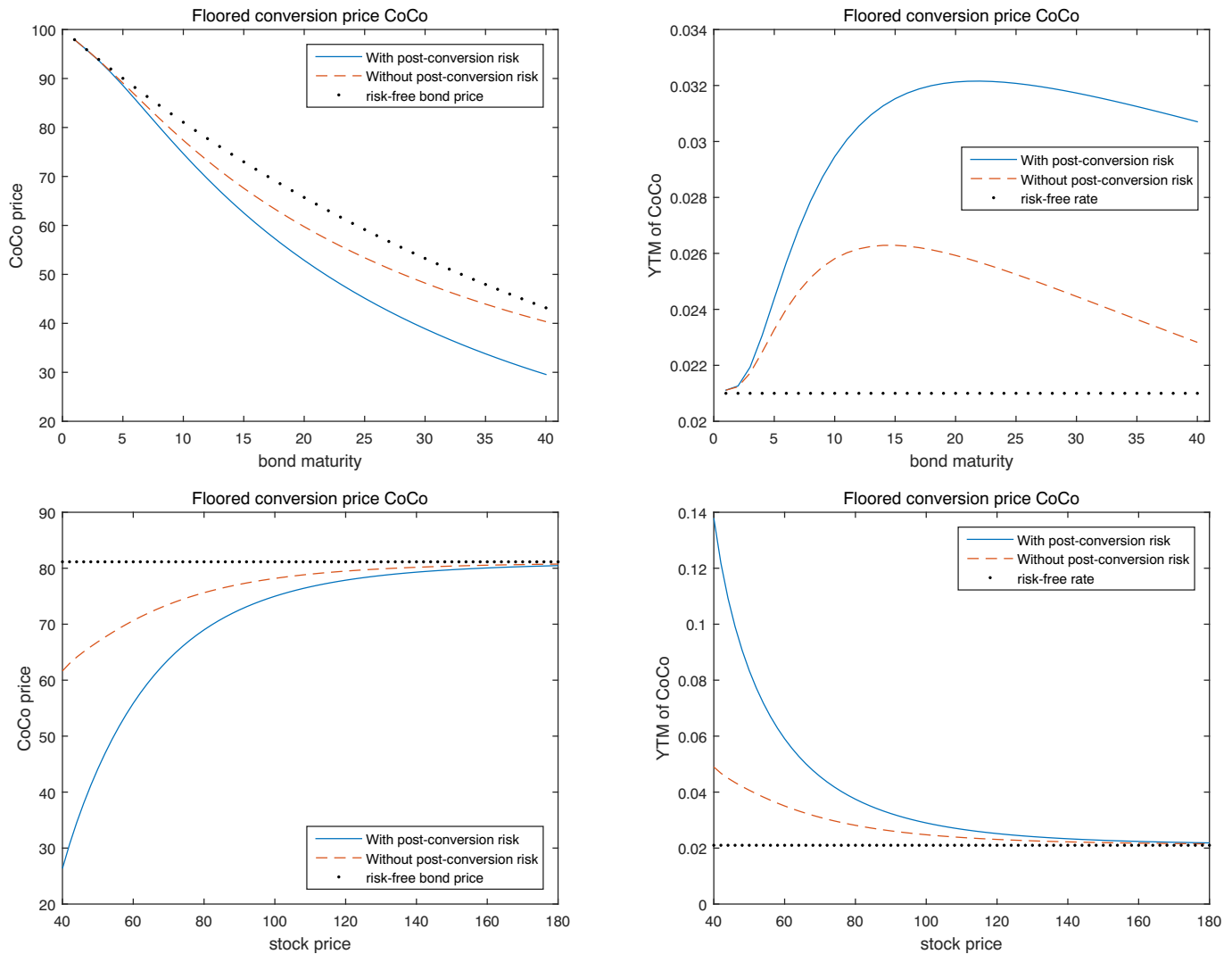


Fig. C5. Comparison of CoCo prices (left) and their YTM (right) for floor conversion prices with and without post-conversion risk with a baseline of a risk-free bond price against maturities (top) and equity prices (bottom).

C.4. Impact of post-conversion risk

We simulate EC CoCo values and their YTM with respect to regulatory default triggers, α_1 , and compute the amounts of $\mathcal{P}(\alpha_1)$ and $\mathcal{P}(\alpha_1)$ derived in Eqs. (22) and (24), respectively. Fig. C6 displays the CoCo values (left) and YTM (right) with a fixed conversion price and a conversion trigger level $\alpha_0 = 7\%$ when α_0 is between 0% and 7%. The figures explain two extreme cases: The end points in Fig. C6 are the extreme cases. From the regulatory perspective, the left end points with $\alpha_1 = 0\%$ mean the case when a CoCo-issuing bank never default before maturity after EC, and the right end points with $\alpha_1 = 7\%$ mean the case when an CoCo-issuing bank defaults immediately EC occurs.

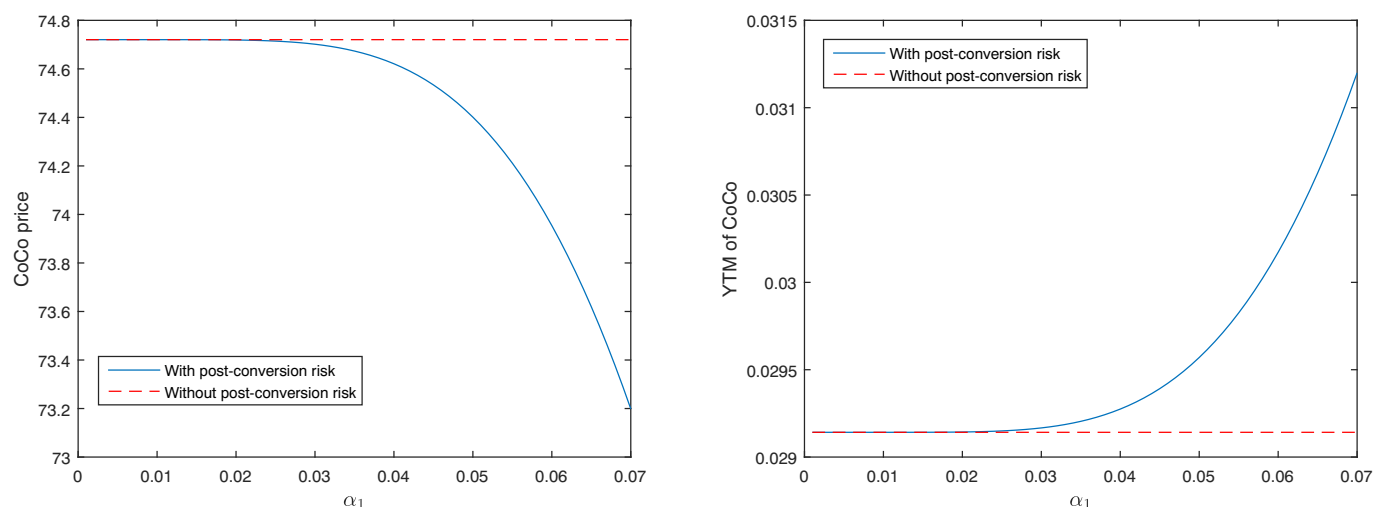


Fig. C6. Comparison of the CoCo prices (left) and CoCo YTM (right) with/without post-conversion risk with respect to regulatory default level α_1 .

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